ABLEITUNGEN

Ableitungsregeln:

Potenzregel:	$f(x)=x^n$	$f'(x)=n\cdot x^{n-1}$
Summenregel:	$f(x)=g(x)+h(x)$	$f'(x)=g'(x)+h'(x)$
Faktorregel:	$f(x)=c\cdot g(x)$	$f'(x)=c\cdot g'(x)$
Kettenregel:	$f(x)=u(v(x))$	$f'(x)=u'(v(x))\cdot v'(x)$
Produktregel:	$f(x)=u(x)\cdot v(x)$	$f'(x)=u'(x)\cdot v(x)+u(x)\cdot v'(x)$
Quotientenregel:	$f(x)=\frac{u(x)}{v(x)}$	$f'(x)=\frac{u'(x)\cdot v(x)-u(x)\cdot v'(x)}{v^2(x)}$

Ableitungen elementarer Funktionen:

Funktion	Ableitung
$f(x)=c$	$f'(x)=0$
$f(x)=x$	$f'(x)=1$
$f(x)=\lvert x\rvert$	$f'(x)=\frac{\lvert x\rvert}{x}$
$f(x)=x^n$	$f'(x)=nx^{n-1}$
$f(x)=\frac{1}{x^n}$	$f'(x)=-\frac{n}{x^{n+1}}$
$f(x)=\sqrt{x}$	$f'(x)=\frac{1}{2\sqrt{x}}$
$f(x)=\sqrt[n]{x}$	$f'(x)=\frac{1}{n\sqrt[n]{x^{n-1}}}$
$f(x)=\sin x$	$f'(x)=\cos x$
$f(x)=\cos x$	$f'(x)=-\sin x$
$f(x)=\ln x$	$f'(x)=\frac{1}{x}$
$f(x)=\ln\lvert x\rvert$	$f'(x)=\frac{1}{x}$
$f(x)=\log_a x$	$f'(x)=\frac{1}{x\ln a}; a>0; a\neq 1$
$f(x)=\tan x$	$f'(x)=\frac{1}{\cos^2 x}$
$f(x)=a^x$	$f'(x)=a^x\cdot\ln a$
$f(x)=e^x$	$f'(x)=e^x$

Ableitungen verketteter Funktionen:

Funktion	Ableitung
$f(u)=u^n\,,\; n\in \mathbb{Z}$	$f'(u)=nu^{n-1}\cdot u'$
$f(u)=\lvert u\rvert$	$f'(u)=\frac{\lvert u\rvert}{u}\cdot u'$
$f(u)=\sqrt{u}$	$f'(u)=\frac{u'}{2\sqrt{u}}$
$f(u)=\sin u$	$f'(u)=\cos u\cdot u'$
$f(u)=\cos u$	$f'(u)=-\sin u\cdot u'$
$f(u)=\ln\lvert u\rvert$	$f'(u)=\frac{u'}{u}$
$f(u)=\tan u$	$f'(u)=\frac{u'}{\cos^2 u}$
$f(u)=a^u$	$f'(u)=a^u\cdot u'\cdot\ln a$
$f(u)=e^u$	$f'(u)=e^u\cdot u'$
$f(u)=u^v$	$f'(u)=u^v\cdot v'\cdot\ln u+v\cdot u^{v-1}\cdot u'$

1. $f(x) = \frac{2}{x^2+3} = 2 \cdot \left(x^2+3\right)^{-1}$.

$f'(x) = 2 \cdot (-1) \cdot \left(x^2+3\right)^{-2} \cdot 2x = -4x \cdot \left(x^2+3\right)^{-2} = -\frac{4x}{\left(x^2+3\right)^2}$.

$\underline{f'(x) = -\frac{4x}{\left(x^2+3\right)^2}}$.

2. $f(x) = xe^{2x}$.

$f'(x) = 1 \cdot e^{2x} + x \cdot e^{2x} \cdot 2 = (1+2x) \cdot x^2 \cdot e^{2x}$.

$\underline{f'(x) = (1+2x) \cdot x^2 \cdot e^{2x}}$.

3. $f(x) = \frac{1}{2} \cdot \sin(2x^2)$.

$f'(x) = \frac{1}{2} \cdot \cos(2x^2) \cdot 4x = 2x \cdot \cos(2x^2)$.

$\underline{f'(x) = 2x \cdot \cos(2x^2)}$.

4. $f(x) = (2+\sin x)^3$.

$\underline{f'(x) = 3 \cdot (2+\sin x)^2 \cdot \cos x}$.

5. $f(x) = x^3 \cdot \sin(2x+1)$.

$\underline{f'(x) = 3x^2 \cdot \sin(3x+1) + x^3 \cdot \cos(2x+1) \cdot 2}$.

6. $f(x) = \frac{1}{x} + 1$.

$f(x) = x^{-1} + 1$.

$f'(x) = (-1) \cdot x^{-2} = -x^{-2} = -\frac{1}{x^2}$.

$\underline{f'(x) = -\frac{1}{x^2}}$.

7. $f(x) = 3 - \frac{3}{x^2}$.

$f(x) = 3 - 3 \cdot x^{-2}$.

$f'(x) = -3 \cdot (-2) \cdot x^{-3} = 6 \cdot x^{-3} = \frac{6}{x^3}$.

$\underline{f'(x) = \frac{6}{x^3}}$.

8. $F(t) = -\frac{2}{e^t + 3}$

$F(t) = -2 \cdot (e^t + 3)^{-1}$

$F'(t) = -2 \cdot (-1) \cdot (e^t + 3)^{-2} \cdot e^t = \frac{2e^t}{(e^t + 1)^2}$.

$\underline{F'(t) = \frac{2e^t}{(e^t + 1)^2}}$.

9. $f(x) = 3 \cdot \sin\left(\frac{\pi}{6} x\right)$

$f'(x) = 3 \cdot \frac{\pi}{6} \cdot \cos\left(\frac{\pi}{6} x\right) = \frac{\pi}{2} \cdot \cos\left(\frac{\pi}{6} x\right)$.

$\underline{f'(x) = \frac{\pi}{2} \cdot \cos\left(\frac{\pi}{6} x\right)}$.

10. $f(t) = 10t \cdot e^{-0,25t}$

$f'(t) = 10(1 \cdot e^{-0,25t} + t \cdot (-0,25) \cdot e^{-0,25t}) = 10 \cdot (1 - 0,25t) \cdot e^{-0,25t}$.

$\underline{f'(t) = 10 \cdot (1 - 0,25t) \cdot e^{-0,25t}}$.

11. $g(x) = 310 - (2x^2 + 35x + 310) \cdot e^{-0,12x}$.

$$g'(x) = -(4x+35) \cdot e^{-0,12x} - (2x^2 + 35x + 310) \cdot (-0,12) \cdot e^{-0,12x}$$

$$= (-4x - 35 + 0,24x^2 + 4,2x + 37,2) \cdot e^{-0,12x} = \left(0,24x^2 + 0,2x + 2,2\right) \cdot e^{-0,12x} \text{.}$$

$$\underline{g'(x) = \left(0,24x^2 + 0,2x + 2,2\right) \cdot e^{-0,12x}} \text{.}$$

12. $f(x) = 12 \cdot \sin\left[\frac{\pi}{8}(x-7)\right] + 15$.

$$f'(x) = 12 \cdot \frac{\pi}{8} \cdot \cos\left[\frac{\pi}{8}(x-7)\right] = \frac{3\pi}{2} \cdot \cos\left[\frac{\pi}{8}(x-7)\right].$$

$$\underline{f'(x) = \frac{3\pi}{2} \cdot \cos\left[\frac{\pi}{8}(x-7)\right]}.$$

13. $h(x) = 10 \cdot \sin\left[\frac{\pi}{2}(x-5,5)\right] + 2ax + b$.

$$h'(x) = 10 \cdot \frac{\pi}{2} \cdot \cos\left[\frac{\pi}{2}(x-5,5)\right] + 2a = 5\pi \cdot \cos\left[\frac{\pi}{2}(x-5,5)\right] + 2a \text{.}$$

$$\underline{h'(x) = 5\pi \cdot \cos\left[\frac{\pi}{2}(x-5,5)\right] + 2a}\text{.}$$

14. $f(t) = 1200 - 900 \cdot e^{-0,01 \cdot t}$.

$$f'(t) = -900 \cdot (-0,01) \cdot e^{-0,01 \cdot t} = 9 \cdot e^{-0,01 \cdot t} \text{.}$$

$$\underline{f'(t) = 9 \cdot e^{-0,01 \cdot t}}\text{.}$$

15. $f(x) = 5 - \frac{50}{(x^2 - 10)^2}$.

$$f(x) = 5 - 50 \cdot \left(x^2 - 10\right)^{-2}$$

$$\underline{f'(x) = -50 \cdot (-2) \cdot \left(x^2 - 10\right)^{-3} \cdot 2x = \frac{200x}{(x^2 - 10)^3}}\text{.}$$

16. $f(x)=\sqrt{x^2+1}$.

$$f(x)=\left(x^2+1\right)^{\frac{1}{2}}$$
$$f'(x)=\frac{1}{2}\cdot\left(x^2+1\right)^{-\frac{1}{2}}\cdot 2x = x\cdot\left(x^2+1\right)^{-\frac{1}{2}}=\frac{x}{\left(x^2+1\right)^{\frac{1}{2}}}=\frac{x}{\sqrt{x^2+1}}\ .$$

$$\underline{f'(x)=\frac{x}{\sqrt{x^2+1}}}\,.$$

17. $f(x)=\dfrac{1}{e^{\frac{1}{x}}\cdot x^2}$.

$$f(x)=e^{-\frac{1}{x}}\cdot x^{-2}.$$

$$f'(x)=e^{-\frac{1}{x}}\cdot\frac{1}{x^2}\cdot x^{-2}+e^{-\frac{1}{x}}\cdot(-2)\cdot x^{-3}=e^{-\frac{1}{x}}\cdot\frac{1}{x^4}-2e^{-\frac{1}{x}}\cdot\frac{1}{x^3}$$
$$=\left(\frac{1}{x^4}-\frac{2}{x^3}\right)\cdot e^{-\frac{1}{x}}.$$

$$\underline{f'(x)=\left(\frac{1}{x^4}-\frac{2}{x^3}\right)\cdot e^{-\frac{1}{x}}}\,.$$

18. $f(x)=\sqrt{e^{3x}}$

$$f(x)=\left(e^{3x}\right)^{\frac{1}{2}}=e^{\frac{3}{2}x}$$

$$\underline{f'(x)=\frac{3}{2}\cdot e^{\frac{3}{2}x}}\,.$$

19. $f(x)=\frac{1}{3}\cdot\sin^3(x^3)$.

$$f(x)=\frac{1}{3}\cdot\left(\sin\left(x^3\right)\right)^3.$$

$$f'(x)=\frac{1}{3}\cdot 3\cdot\sin^2(x^3)\cdot\cos\left(x^3\right)\cdot 3x^2=3x^2\cdot\sin^2(x^3)\cdot\cos\left(x^3\right).$$

$$\underline{f'(x)=3x^2\cdot\sin^2(x^3)\cdot\cos\left(x^3\right)}.$$

20. $f(x) = x \cdot e^x \cdot \cos x$.

$$f'(x) = 1 \cdot e^x \cdot \cos x + x \cdot e^x \cdot \cos x + x \cdot e^x \cdot (-\sin x)$$
$$= (\cos x + x \cdot \cos x - x \cdot \sin x) \cdot e^x.$$

$$\underline{f'(x) = (\cos x + x \cdot \cos x - x \cdot \sin x) \cdot e^x.}$$

21. $f(x) = \sqrt[3]{\sqrt{x}}$.

$$f(x) = \sqrt[3]{\sqrt{x}} = \left(x^{\frac{1}{2}}\right)^{\frac{1}{3}} = x^{\frac{1}{2}\cdot\frac{1}{3}} = x^{\frac{1}{6}}.$$

$$f'(x) = \frac{1}{6} \cdot x^{\frac{1}{6}-1} = \frac{1}{6} \cdot x^{-\frac{5}{6}} = \frac{1}{6 \cdot x^{\frac{5}{6}}} = \frac{1}{6 \cdot \sqrt[6]{x^5}}.$$

$$\underline{f'(x) = \frac{1}{6 \cdot \sqrt[6]{x^5}}.}$$