Ableitungen

Lösungen:

1. $f(x) = (3x-2)\cdot e^{-3x}$

$$f'(x) = 3\cdot e^{-3x} + (3x-2)\cdot(-3)\cdot e^{-3x} = \big(3 + (3x-2)\cdot(-3)\big)\cdot e^{-3x}$$

$$= (3 - 9x + 6)\cdot e^{-3x} = (9-9x)\cdot e^{-3x} = 9\cdot(1-x)\cdot e^{-3x}.$$

$$\underline{f'(x) = 9\cdot(1-x)\cdot e^{-3x}}.$$

2. $f(x) = \dfrac{5x}{x^2+8}$

$$f'(x) = \frac{5\cdot(x^2+8) - 5x\cdot 2x}{(x^2+8)^2} = \frac{5x^2+40-10x^2}{(x^2+8)^2} = \frac{-5x^2+40}{(x^2+8)^2} = \frac{-5(x^2-8)}{(x^2+8)^2}.$$

$$\underline{f'(x) = \frac{-5(x^2-8)}{(x^2+8)^2}}.$$

3. $f(x) = \dfrac{x^{-2}}{e^x} = x^{-2}\cdot e^{-x}$

$$f'(x) = -2\cdot x^{-3}\cdot e^{-x} + x^{-2}\cdot e^{-x}\cdot(-1) = -\frac{2}{x^3}\cdot e^{-x} - \frac{1}{x^2}\cdot e^{-x} = -\left(\frac{2}{x^3}+\frac{1}{x^2}\right)\cdot e^{-x}$$

$$= -\frac{2+x}{x^3}\cdot e^{-x} = -\frac{x+2}{x^3\cdot e^x}.$$

$$\underline{f'(x) = -\frac{x+2}{x^3\cdot e^x}}.$$

4. $f(x) = e^{2x}\cdot\cos(0{,}5x-1)$

$$f'(x) = 2\cdot e^{2x}\cdot\cos(0{,}5x-1) + e^{2x}\cdot(-\sin(0{,}5x-1)\cdot 0{,}5$$

$$= (2\cdot\cos(0{,}5x-1) - 0{,}5\cdot\sin(0{,}5x-1))\cdot e^{2x}$$

$$\underline{f'(x) = (2\cdot\cos(0{,}5x-1) - 0{,}5\cdot\sin(0{,}5x-1))\cdot e^{2x}}.$$

5. $f(x)=\frac{2-e^{3x}}{e^x}=\frac{2}{e^x}-\frac{e^{3x}}{e^x}=2e^{-x}-e^{2x}$

$$f'(x)=2\cdot(-1)\cdot e^{-x}-2\cdot e^{2x}=-2e^{-x}-2e^{2x}=-\frac{2}{e^x}-2e^{2x}.$$

$$\underline{f'(x)=-\frac{2}{e^x}-2e^{2x}.}$$

6. $f'(x)=\frac{2x\cdot(3x^2-2)-(x^2-1)\cdot 6x}{(3x^2-2)^2}=\frac{6x^3-4x-6x^3+6x}{(3x^2-2)^2}=\frac{2x}{(3x^2-2)^2}.$

$$\underline{f'(x)=\frac{2x}{(3x^2-2)^2}.}$$

7. $f'(x)=\frac{1}{3}\cdot 1\cdot\sin(3x^2-1)+\frac{1}{3}x\cdot\cos(3x^2-1)\cdot 6x=\frac{1}{3}\cdot\sin(3x^2-1)+2x^2\cdot\cos(3x^2-1)$

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$$\underline{f'(x)=\frac{1}{3}\cdot\sin(3x^2-1)+2x^2\cdot\cos(3x^2-1).}$$

8. $f(x)=\sqrt{2x}+\frac{1}{2x^2-1}=\sqrt{2}\cdot x^{\frac{1}{2}}+(2x^2-1)^{-1}$

$$f'(x)=\sqrt{2}\cdot\frac{1}{2}\cdot x^{-\frac{1}{2}}+(-1)\cdot(2x^2-1)^{-2}\cdot 4x=\frac{\sqrt{2}}{2}\cdot x^{-\frac{1}{2}}-4x\cdot(2x^2-1)^{-2}$$

$$=\frac{1}{\sqrt{2x}}-\frac{4x}{(2x^2-1)^2}.$$

9. $f(x)=\frac{\sin\left(e^x\right)}{e^x}$

$$f'(x)=\frac{\cos\left(e^x\right)\cdot e^x\cdot e^x-\sin\left(e^x\right)\cdot e^x}{(e^x)^2}=\frac{\cos\left(e^x\right)\cdot e^{2x}-\sin\left(e^x\right)\cdot e^x}{e^{2x}}=\cos\left(e^x\right)-\frac{\sin\left(e^x\right)}{e^x}.$$

$$\underline{f'(x)=\cos\left(e^x\right)-\frac{\sin\left(e^x\right)}{e^x}.}$$

10. $f(x)=\left(2+e^{-3x}\right)^2$

$$f'(x)=2\cdot\left(2+e^{-3x}\right)\cdot e^{-3x}\cdot(-3)=-6\cdot\left(2+e^{-3x}\right)\cdot e^{-3x}$$

$$\underline{f'(x)=-6\cdot\left(2+e^{-3x}\right)\cdot e^{-3x}..}$$

11. $f(x)=\sqrt{e^{3x}}=e^{\frac{3}{2}x}$.

$$\underline{f'(x)=\frac{3}{2}\cdot e^{\frac{3}{2}x}}.$$

12. $f(x)=\frac{1}{3}\sin^3(x^3)$.

$$f'(x)=\frac{1}{3}\cdot 3\cdot 3x^2\cdot\sin^2(x^3)=3x^2\cdot\sin^2(x^3).$$

$$\underline{f'(x)=3x^2\cdot\sin^2(x^3)}.$$

13. $f(x)=\sqrt[3]{\sqrt{x}}=\left(x^{\frac{1}{2}}\right)^{\frac{1}{3}}=x^{\frac{1}{2}\cdot\frac{1}{3}}=x^{\frac{1}{6}}$.

$$f'(x)=\frac{1}{6}\cdot x^{\frac{1}{6}-1}=\frac{1}{6}\cdot x^{-\frac{5}{6}}=\frac{1}{6\cdot x^{\frac{5}{6}}}=\frac{1}{6\cdot\sqrt[6]{x^5}}.$$

$$\underline{f'(x)=\frac{1}{6\cdot\sqrt[6]{x^5}}}.$$

14. $f(x)=\frac{\ln x}{x},\ x>0$

$$f'(x)=\frac{\frac{1}{x}\cdot x-\ln x\cdot 1}{x^2}=\frac{1-\ln x}{x^2}.$$

$$\underline{f'(x)=\frac{1-\ln x}{x^2}}.$$

15. $g(x)=\frac{x^5}{2x^6+7}$.

$$g'(x)=\frac{5x^4\cdot(2x^6+7)-x^5\cdot 12x^5}{(2x^6+7)^2}$$

$$=\frac{10x^{10}+35x^4-12x^{10}}{(2x^6+7)^2}=\frac{35x^4-2x^{10}}{(2x^6+7)^2}.$$

$$\underline{g'(x)=\frac{35x^4-2x^{10}}{(2x^6+7)^2}}.$$

16. $h(x)=5\cdot\sin\left[\frac{\pi}{2}(x-8)\right]+2ax+b$

$$h'(x)=5\cdot\frac{\pi}{2}\cdot\cos\left[\frac{\pi}{2}(x-8)\right]+2a=\frac{5\pi}{2}\cdot\cos\left[\frac{\pi}{2}(x-8)\right]+2a.$$

$$\underline{h'(x)=\frac{5\pi}{2}\cdot\cos\left[\frac{\pi}{2}(x-8)\right]+2a}.$$

17. $f(t) = -\dfrac{1}{e^{2t}+1} = -(e^{2t}+1)^{-1}$

$$f'(t) = -(-1)\cdot(e^{2t}+1)^{-2}\cdot e^{2t}\cdot 2 = \frac{2e^{2t}}{(e^{2t}+1)^2}\,.$$

$$\underline{f'(t) = \frac{2e^{2t}}{(e^{2t}+1)^2}\,.}$$

18. $f_t(x) = 3t - \dfrac{4t}{x^2} = 3t - 4t\cdot x^{-2}$

$$f_t'(x) = -4t\cdot(-2)\cdot x^{-3} = 8t\cdot x^{-3} = \frac{8t}{x^3}\,.$$

$$\underline{f_t'(x) = \frac{8t}{x^3}\,.}$$

19. $f(x) = (x+\sin x)^2$

$$\underline{f'(x) = 2(x+\sin x)\cdot(1+\cos x)}$$

20. $f(x) = x\cdot e^x\cdot\cos x$

$$f'(x) = \left(x\cdot e^x\right)'\cdot\sin x + x\cdot e^x\cdot\cos x$$
$$= \left(1\cdot e^x + x\cdot e^x\right)\cdot\sin x + x\cdot e^x\cdot\cos x$$
$$= e^x\cdot\sin x + x\cdot e^x\cdot\sin x + x\cdot e^x\cdot\cos x$$
$$= (\sin x + x\cdot\sin x + x\cdot\cos x)\cdot e^x.$$

$$\underline{f'(x) = (\sin x + x\cdot\sin x + x\cdot\cos x)\cdot e^x.}$$