

# Stammfunktionen

## Lösungen:

1.  $f(x) = \frac{1}{x^3} + \cos(2x) = x^{-3} + \cos(2x)$

$$F(x) = \frac{1}{-2} \cdot x^{-2} + \frac{1}{2} \cdot \sin(2x) = -\frac{1}{2x^2} + \frac{1}{2} \cdot \sin(2x)$$

2.  $f(x) = 3 \cdot \sin\left(\frac{1}{3}x\right) - \frac{1}{2}x^2$

$$F(x) = -3 \cdot \frac{1}{\frac{1}{3}} \cdot \cos\left(\frac{1}{3}x\right) - \frac{1}{2} \cdot \frac{1}{3}x^3 = -9 \cdot \cos\left(\frac{1}{3}x\right) - \frac{1}{6}x^3$$

3.  $f(x) = \frac{1}{\sqrt{x}} + 4x^{-3} = x^{-\frac{1}{2}} + 4x^{-3}$

$$F(x) = \frac{1}{\frac{1}{2}} \cdot x^{\frac{1}{2}} + 4 \cdot \frac{1}{-2}x^{-2} = 2x^{\frac{1}{2}} - 2x^{-2} = 2\sqrt{x} - 2x^{-2}$$

4.  $f(x) = 2(x^3 + 3e^{-3x})$

$$F(x) = 2\left(\frac{1}{4}x^4 + 3 \cdot \frac{1}{-3}e^{-3x}\right) = 2\left(\frac{1}{4}x^4 - e^{-3x}\right)$$

5.  $f(x) = \left(3 - \frac{1}{2}x\right)^6$

$$F(x) = \frac{1}{-\frac{1}{2}} \cdot \frac{1}{7} \left(3 - \frac{1}{2}x\right)^7 = -2 \cdot \frac{1}{7} \left(3 - \frac{1}{2}x\right)^7 = -\frac{2}{7} \left(3 - \frac{1}{2}x\right)^7$$

6.  $f(x) = 2\sqrt{x} - 3\cos\left(\frac{1}{3}x - 1\right) = 2x^{\frac{1}{2}} - 3\cos\left(\frac{1}{3}x - 1\right)$

$$F(x) = \frac{1}{\frac{3}{2}}x^{\frac{3}{2}} - 3 \cdot \frac{1}{\frac{1}{3}}\sin\left(\frac{1}{3}x - 1\right) = \frac{2}{3}\sqrt{x^3} - 9\sin\left(\frac{1}{3}x - 1\right)$$

7.  $f(x) = \frac{1}{x} - \pi \cdot \cos(\pi x)$

$$F(x) = \ln|x| - \pi \cdot \frac{1}{\pi} \cdot \sin(\pi x) = \ln|x| - \sin(\pi x)$$

$$8. f(x) = \sqrt{2x+5} = (2x+5)^{\frac{1}{2}}$$

$$F(x) = \frac{1}{2} \cdot \frac{2}{3} \cdot (2x+5)^{\frac{3}{2}} = \frac{1}{3} (2x+5)^{\frac{3}{2}} = \frac{1}{3} \sqrt{(2x+5)^3}$$

$$9. f(x) = \frac{1}{2x-1} + 4 \cdot \cos(2x-1)$$

$$F(x) = \frac{1}{2} \cdot \ln|2x-1| + 2 \cdot \sin(2x-1)$$

$$10. f(x) = e^{\frac{1}{3}x} + \frac{1}{2x} + 3$$

$$F(x) = \frac{1}{\frac{1}{3}} e^{\frac{1}{3}x} + \frac{1}{2} \ln|x| + 3x = 3e^{\frac{1}{3}x} + \frac{1}{2} \ln|x| + 3x$$

$$11. f(x) = \frac{2}{x^2} - \frac{x^2}{2} = 2x^{-2} - \frac{1}{2}x^2$$

$$F(x) = 2 \cdot \frac{1}{-1} x^{-1} - \frac{1}{2} \cdot \frac{1}{3} x^3 = -\frac{2}{x} - \frac{1}{6} x^3$$

$$12. f(x) = \frac{2}{(4x-3)^3} = 2 \cdot (4x-3)^{-3}$$

$$F(x) = 2 \cdot \frac{1}{4} \cdot \frac{1}{-2} (4x-3)^{-2} = -\frac{1}{4} (4x-3)^{-2} = -\frac{1}{(4x-3)^2}$$

$$13. f(x) = \frac{1}{3x^2} + 3 \cdot \sin(5x+1) = \frac{1}{3} \cdot x^{-2} + 3 \cdot \sin(5x+1)$$

$$F(x) = \frac{1}{3} \cdot \frac{1}{-1} x^{-1} - 3 \cdot \frac{1}{5} \cos(5x+1) = -\frac{1}{3x} - \frac{3}{5} \cos(5x+1)$$

$$14. f(x) = 5e^{5x-3} + \frac{1}{e^x} + 3x = 5e^{5x-3} + e^{-x} + 3x$$

$$F(x) = 5 \cdot \frac{1}{5} e^{5x-3} + \frac{1}{-1} \cdot e^{-x} + 3 \cdot \frac{1}{2} x^2 = e^{5x-3} - e^{-x} + \frac{3}{2} x^2$$

$$15. f(x) = \frac{2x^3 - 3x + 1}{x} = \frac{2x^3}{x} - \frac{3x}{x} + \frac{1}{x} = 2x^2 - 3 + \frac{1}{x}$$

$$F(x) = 2 \cdot \frac{1}{3} x^3 - 3x + \ln|x| = \frac{2}{3} x^3 - 3x + \ln|x|$$

$$16. f(x) = \frac{x+1}{2x^3} = \frac{x}{2x^3} + \frac{1}{2x^3} = \frac{1}{2x^2} + \frac{1}{2x^3} = \frac{1}{2} x^{-2} + \frac{1}{2} x^{-3}$$

$$f(x) = \frac{1}{2} \cdot \frac{1}{-1} x^{-1} + \frac{1}{2} \cdot \frac{1}{-2} x^{-2} = -\frac{1}{2x} - \frac{1}{4x^2}$$

$$17. f_t(x) = 7tx^6 - e^{2tx}$$

$$F_t(x) = 7t \cdot \frac{1}{7} x^7 - \frac{1}{2t} \cdot e^{2tx} = tx^7 - \frac{1}{2t} \cdot e^{2tx}$$

$$18. f(x) = \frac{2}{\sqrt{x+3}} = 2(x+3)^{-\frac{1}{2}}$$

$$F(x) = 2 \cdot \frac{1}{\frac{1}{2}} (x+3)^{\frac{1}{2}} = 2 \cdot 2(x+3)^{\frac{1}{2}} = 4\sqrt{x+3}$$

$$19. f(x) = \frac{1}{1-2x} + \sin(\pi)$$

$$F(x) = -\frac{1}{2} \cdot \ln|1-2x| + \sin(\pi) \cdot x$$

$$20. f(x) = 2\pi \sin(2\pi x) - \pi$$

$$F(x) = -2\pi \cdot \frac{1}{2\pi} \cos(2\pi x) - \pi x$$